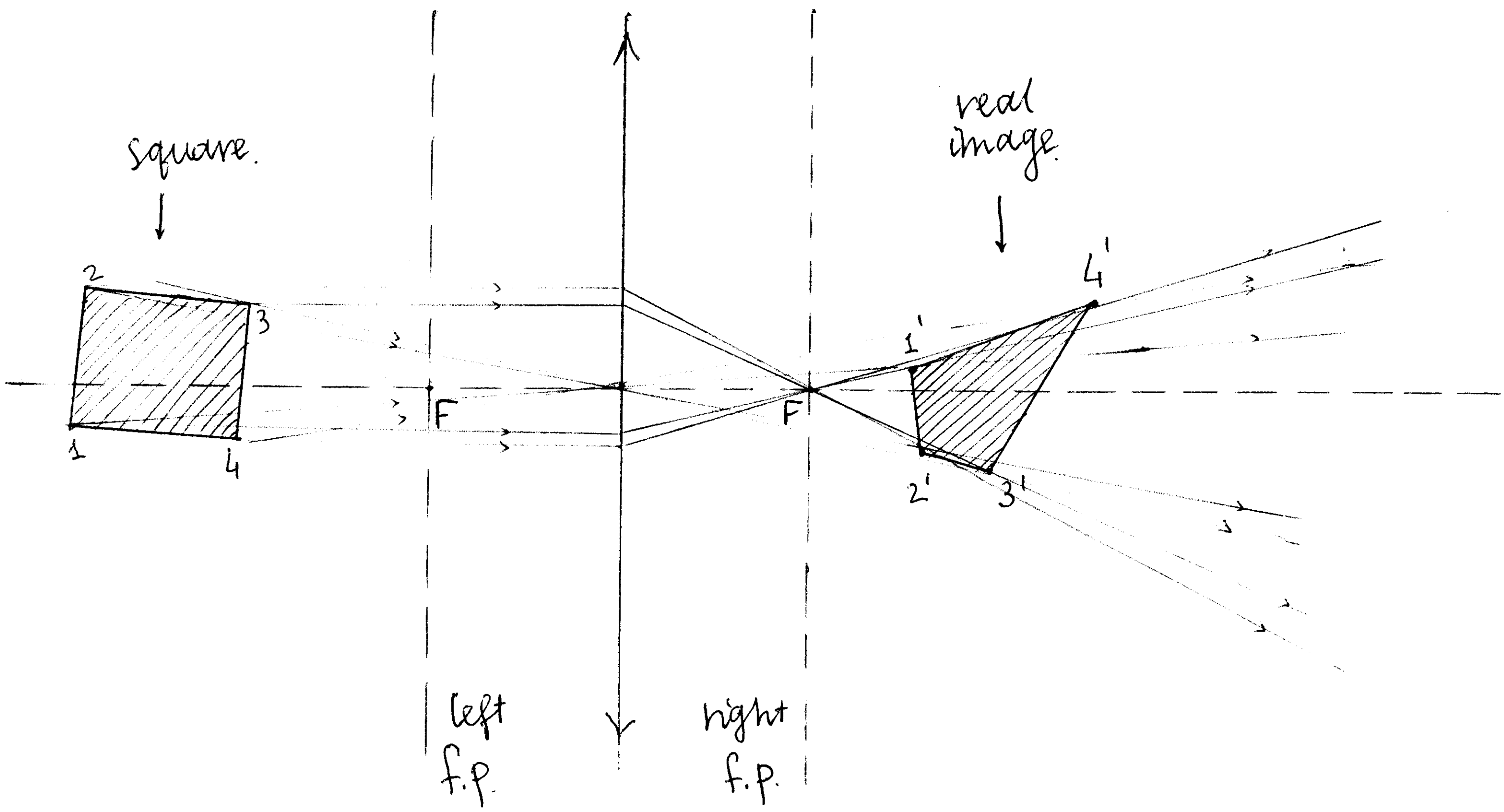
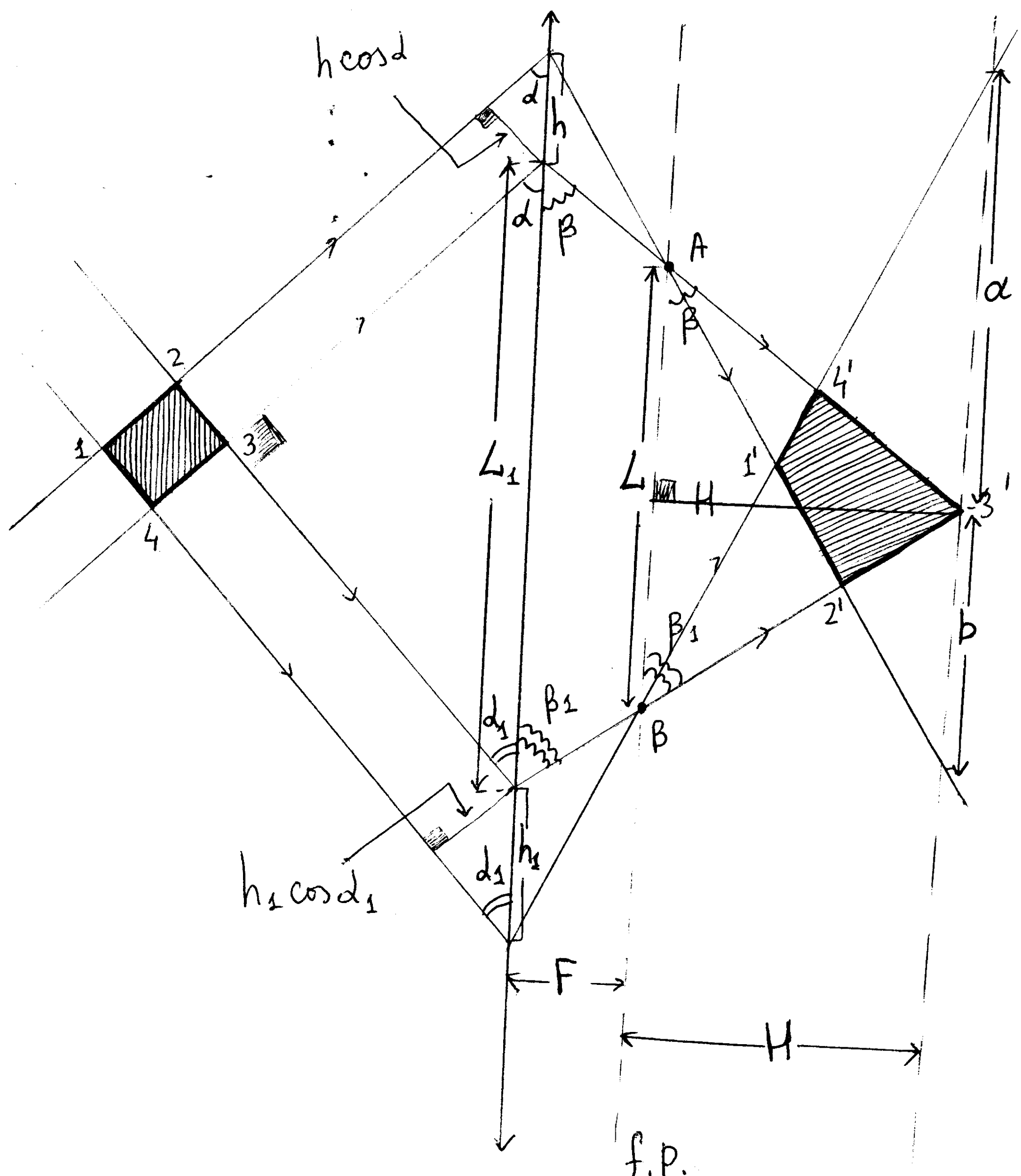
Alexandra Vasileva, Russia.
Problem № 7.

Since the image is real, the only possible configuration of the elements in the problem is the following:



(Both the square and its image are behind the focal planes of the lens; the lens is collecting.)

(14) || (23); (12) || (34) => (1'4') and (2'3'), (1'2') and (3'4') cross in the right focal plane; so it can be reconstructed.

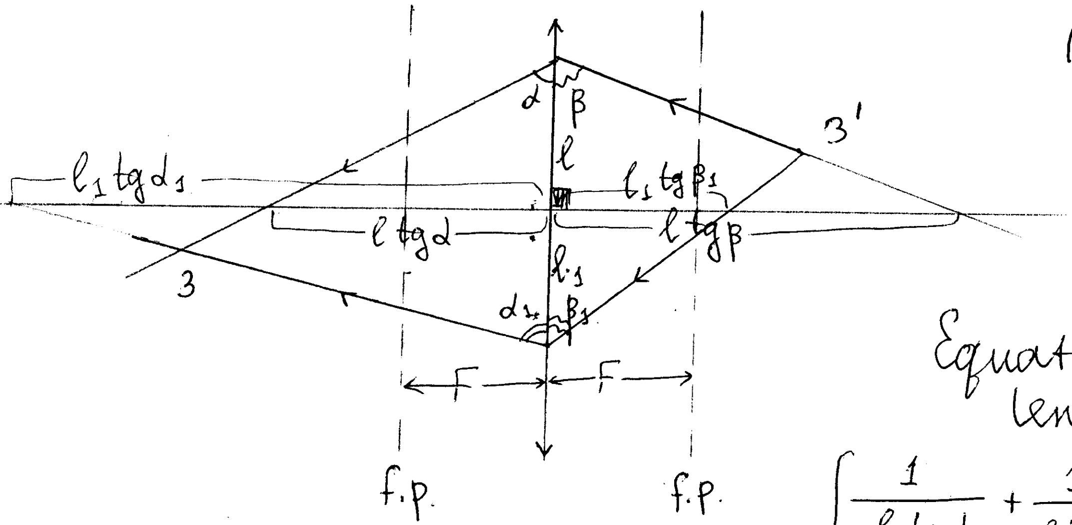


If 1234 is a square, then:

$$\begin{cases} \alpha + \alpha_1 = \frac{\pi}{2}, & (1) \\ h \cos \alpha = h_1 \cos \alpha_1 . & (2) \end{cases}$$

(And if (1) and (2) are true, then 1234 is a square)

((34) and (32))



Equation of a thin lens:

$$\begin{cases} \dfrac{1}{l\,tg\alpha}+\dfrac{1}{l\,tg\beta}=\dfrac{1}{F}, \\ \dfrac{1}{l_1\,tg\alpha_1}+\dfrac{1}{l_1\,tg\beta_1}=\dfrac{1}{F} \end{cases} \Rightarrow$$

$$\Rightarrow \frac{1}{tg\alpha}+\frac{1}{tg\alpha_1}+\frac{1}{tg\beta}+\frac{1}{tg\beta_1}=\frac{l+l_1}{F}=\frac{L_1}{F}=\frac{L\frac{F+H}{H}}{F};$$

$$\frac{1}{tg\beta}+\frac{1}{tg\beta_1}=\frac{L}{H}\ (\text{see } \Delta\, AB3') \Rightarrow$$

$$\Rightarrow \frac{1}{tg\alpha}+\frac{1}{tg\alpha_1}=\frac{LF+LH}{FH}-\frac{LF}{FH}=\frac{LH}{FH}=\frac{L}{F};$$

(1) $\alpha+\alpha_1=\frac{\pi}{2} \Rightarrow tg\alpha_1=\frac{1}{tg\alpha};\ \cos\alpha_1=\sin\alpha \Rightarrow$

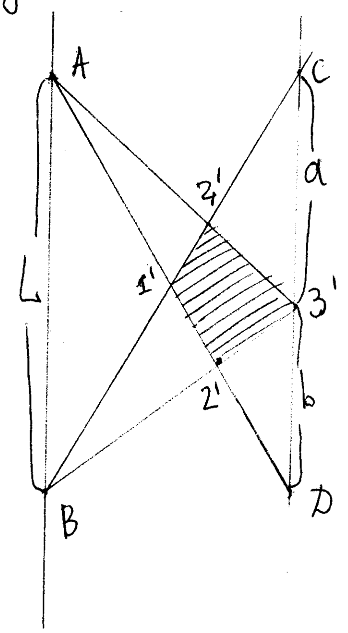
$$\Rightarrow \begin{cases} \dfrac{1}{tg\alpha}+tg\alpha=\dfrac{L}{F}, \\ h\cos\alpha=h_1\sin\alpha \Rightarrow tg\alpha=\dfrac{h}{h_1}. \end{cases} \Rightarrow \frac{L}{F}=\frac{h}{h_1}+\frac{h_1}{h}.$$

$$\frac{F}{H}=\frac{h}{b}=\frac{h_1}{a} \Rightarrow \frac{h}{h_1}=\frac{b}{a} \Rightarrow \frac{L}{F}=\frac{b}{a}+\frac{a}{b}=\frac{a^2+b^2}{ab} \Rightarrow$$

$$\Rightarrow \frac{F}{L}=\frac{ab}{a^2+b^2} \Rightarrow \boxed{F=L\frac{ab}{a^2+b^2}}.$$

This gives us the way to reconstruct the lens.

1.



$(4'3')\cap(1'2')=\{A\}$.

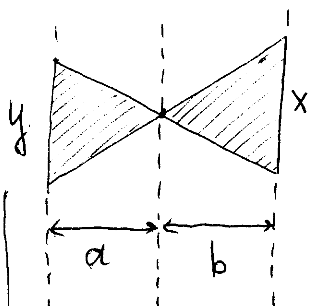
$(4'1')\cap(2'3')=\{B\}$.

(CD): $(CD)\parallel(AB)$;

$\{C\}\in(1'4')$; $\{D\}\in(1'2')$.

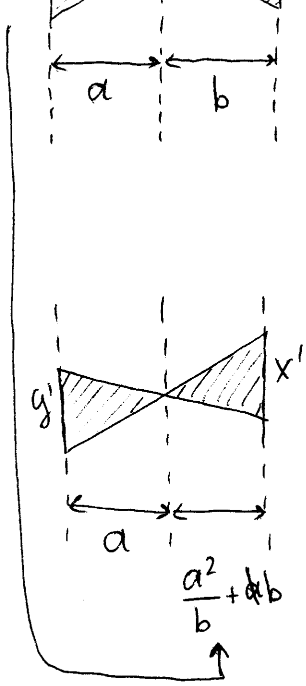
(AB) shows the right focal plane of the lens.

2.



Draw three parallel lines as shown on the figure so that the distances between them are a, b and $a+b$; then, if $x=a$,

$$y = x\frac{a}{b} = \frac{a^2}{b}.$$



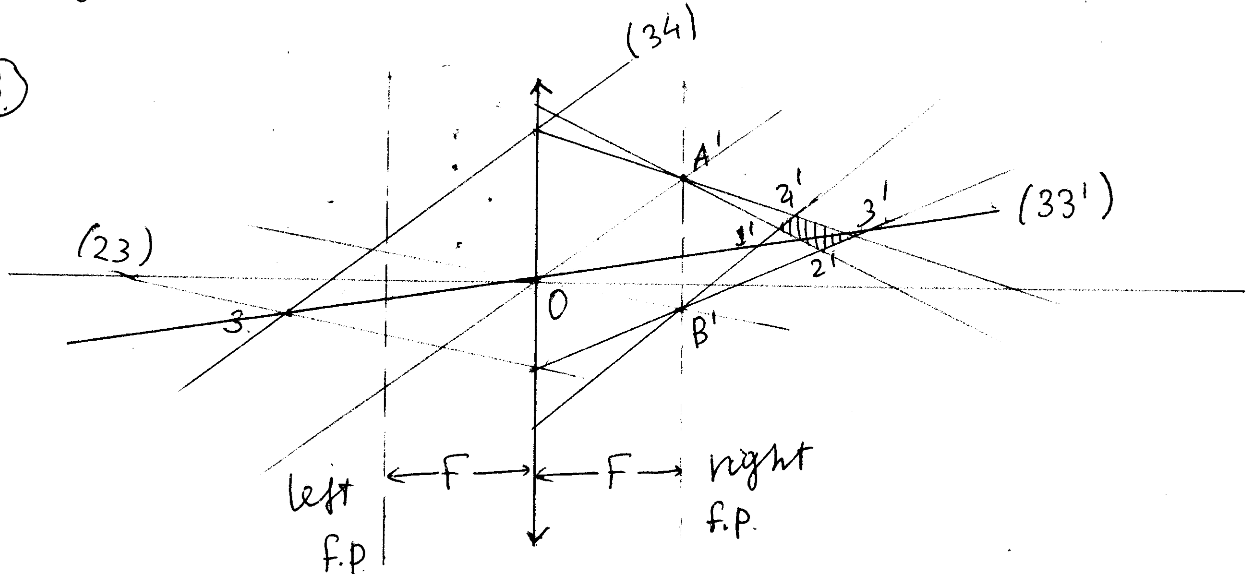
Similarly, $y' = x'\cdot\frac{a}{\frac{a^2}{b}+ab}$;

$$x' = L \Rightarrow y' = L\frac{a}{\frac{a^2}{b}+ab} =$$

$$= L\frac{ab}{a^2+b^2} = F.$$

This way we geometrically built a segment of length F.

3.



Lens: a line parallel to the right f.p, at a ~~with~~ distance F from it

Left f.p.: a line parallel to the lens, at a distance F from it.

$(34) \parallel (OA')$; $(23) \parallel (OB')$ $\Rightarrow$ $(23) \cap (34) = 3$.

$(33') \cap (\text{Lens}) = \{O\}$.

optical centre of the lens.

So the positions of the lens, its optical centre and focuses are defined.

Problem № 7.

Corrections.

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① $h \sin\alpha = h_1 \sin\alpha_1 = h_1 \cos\alpha \Rightarrow tg\,\alpha = \frac{h_1}{h}$, which doesn't affect any further calculations.

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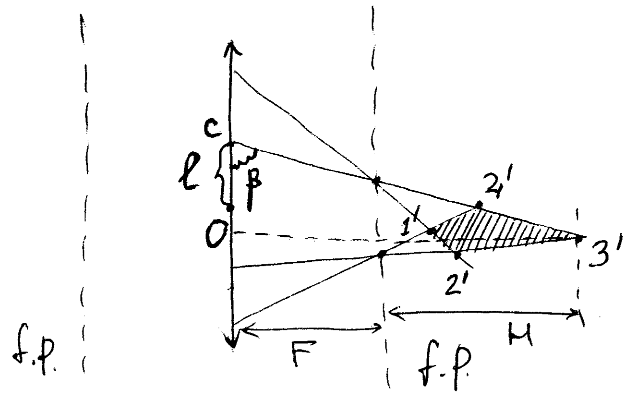
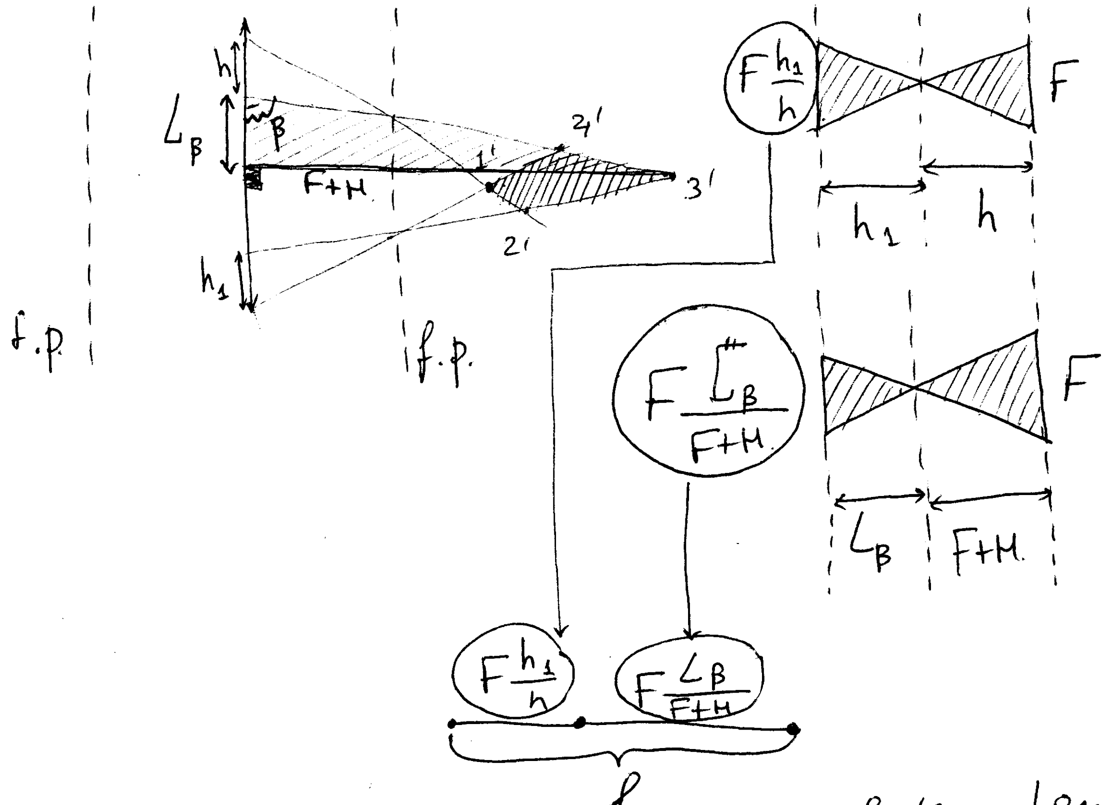
② Correction of the last page:

$$\frac{1}{tg\,\alpha} + \frac{1}{tg\,\beta} = \frac{\ell}{F} \Rightarrow \ell = F\left(\frac{h_1}{h} + \frac{L_\beta}{F+H}\right).$$



$F\frac{h_1}{h}$ and $F\frac{L_\beta}{F+H}$ can be built, using the method described in the solution:



$CO = \ell \Rightarrow O$ (the optical centre of the lens) can be reconstructed.