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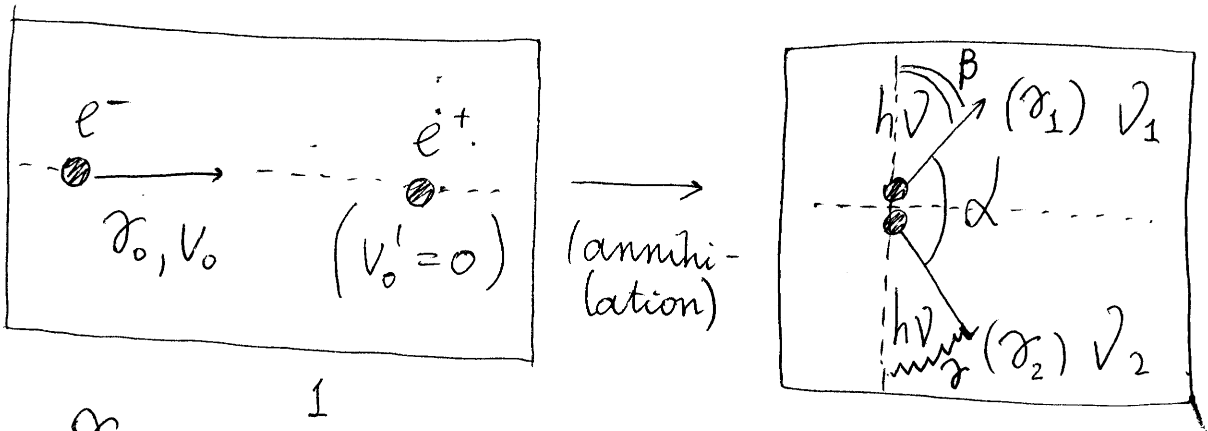
Problem №9



$$\gamma_0 = \frac{1}{\sqrt{1 - V_0^2/c^2}}.$$

Law of conservation of energy for the initial acceleration of e^-:

$$m_0c^2 + eU = m_0c^2 + k m_0c^2 = \gamma_0 m_0 c^2 \Rightarrow \boxed{\gamma_0 = k+1} \Rightarrow$$

$$\Rightarrow \gamma_0^2\left(1 - \frac{V_0^2}{c^2}\right) = 1 = (k+1)^2\left(1 - \frac{V_0^2}{c^2}\right) \Rightarrow$$

$$\Rightarrow 1 - \frac{V_0^2}{c^2} = \frac{1}{(k+1)^2} \Rightarrow \frac{V_0^2}{c^2} = 1 - \frac{1}{(k+1)^2} =$$

$$= \frac{(k+1)^2 - 1}{(k+1)^2} = \frac{k^2 + 2k}{(k+1)^2} \Rightarrow \boxed{V_0 = c\frac{\sqrt{k^2+2k}}{k+1}}.$$

For the annihilation:

$$\begin{cases} m_0c^2 + \gamma_0 m_0 c^2 = (k+2)m_0c^2 = h\nu_1 + h\nu_2 & \text{(Law of cons. of E)} \\ \dfrac{h\nu_1}{c}\cos\beta = \dfrac{h\nu_2}{c}\cos\gamma & \text{(Law of cons. of p, two projections)} \\ \dfrac{h\nu_1}{c}\sin\beta + \dfrac{h\nu_2}{c}\sin\gamma = \gamma_0 m_0 V_0 = m_0 c\sqrt{k^2+2k}. & \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \nu_1 + \nu_2 = \dfrac{m_0c^2}{h}(k+2) \\ \nu_1 = \nu_2 \dfrac{\cos\gamma}{\cos\beta} \\ \nu_1 \sin\beta + \nu_2 \sin\gamma = \dfrac{m_0c^2}{h}\sqrt{k^2+2k} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \nu_2\left(1 + \dfrac{\cos\gamma}{\cos\beta}\right) = \dfrac{m_0c^2}{h}(k+2) \\ \nu_2\left(\dfrac{\cos\gamma\sin\beta}{\cos\beta} + \sin\gamma\right) = \dfrac{m_0c^2}{h}\sqrt{k^2+2k} \end{cases} \qquad \div$$

$$\Downarrow$$

$$\frac{\cos\beta + \cos\gamma}{\sin\beta\cos\gamma + \cos\beta\sin\gamma} = \frac{k+2}{\sqrt{k}\sqrt{k+2}} = \sqrt{\frac{k+2}{k}}$$

$$\Downarrow$$

$$\Downarrow$$

$$\frac{2\cos\frac{\beta+\gamma}{2}\cos\frac{\beta-\gamma}{2}}{2\sin\frac{\beta+\gamma}{2}\cos\frac{\beta+\gamma}{2}}=\sqrt{\frac{k+2}{k}}.$$

$\cos\frac{\beta+\gamma}{2}\neq 0 \quad \left(\frac{\beta+\gamma}{2}\neq\frac{\pi}{2};\ \alpha\neq 0\right).$

$$\Downarrow$$

$$\frac{\cos\frac{\beta-\gamma}{2}}{\sin\frac{\beta+\gamma}{2}}=\sqrt{\frac{k+2}{k}};$$

$(\beta+\gamma)+\alpha=\pi \Rightarrow \frac{\beta+\gamma}{2}+\frac{\alpha}{2}=\frac{\pi}{2}\Rightarrow$

$\Rightarrow \sin\frac{\beta+\gamma}{2}=\cos\frac{\alpha}{2}.$

$$\Downarrow$$

$$\boxed{\cos\frac{\alpha}{2}=\sqrt{\frac{k}{k+2}}\cos\frac{\beta-\gamma}{2}}.$$

$\alpha - \min. \Rightarrow \cos\frac{\alpha}{2}-\max \Rightarrow \cos\frac{\beta-\gamma}{2}-\max\Rightarrow \cos\frac{\beta-\gamma}{2}=1$

$(\beta-\gamma=0\Rightarrow\beta=\gamma)$

$$\Downarrow$$

$$\cos\frac{\alpha_{min}}{2}=\cancel{\cos}\sqrt{\frac{k}{k+2}}\Rightarrow\frac{\alpha_{min}}{2}=\arccos\sqrt{\frac{k}{k+2}}.$$

$$\boxed{\alpha_{min}=2\arccos\sqrt{\frac{k}{k+2}}=(k=1)\ 2\arccos\sqrt{\frac{1}{3}}\simeq 109^\circ}.$$