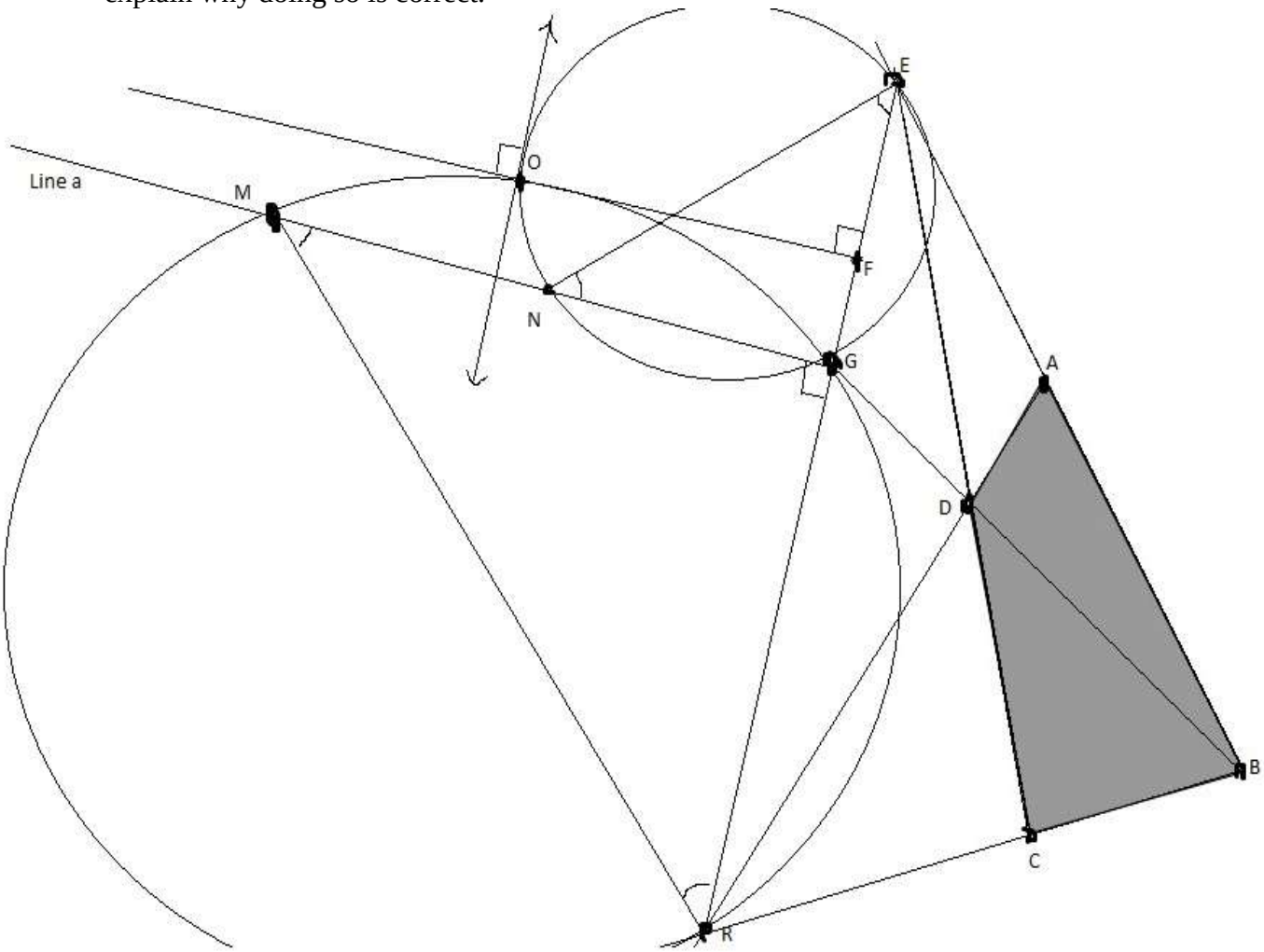
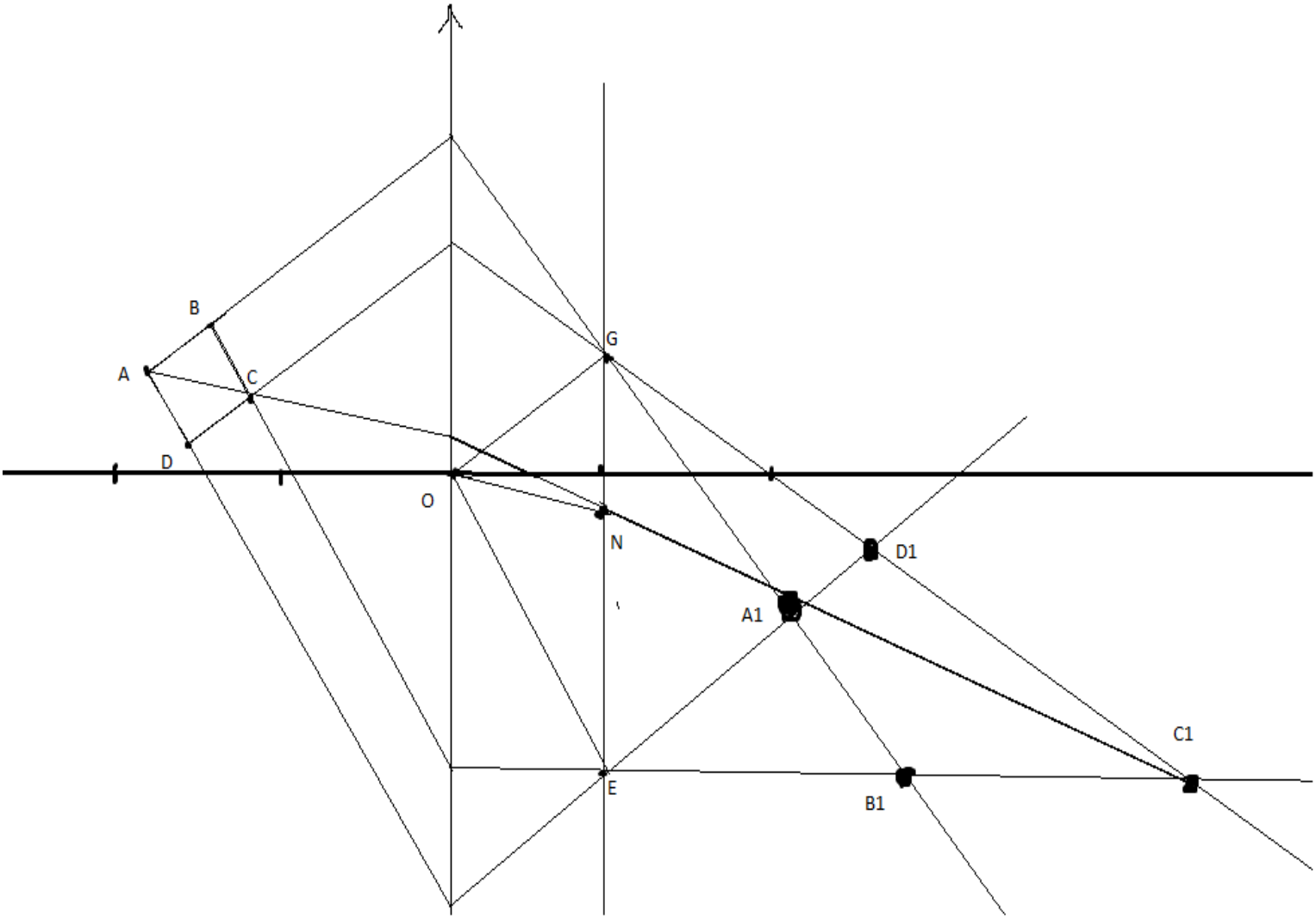
Solution for problem 7 by Ion Toloaca

First I will show how to find the position, orientation and focal distance of the lens, then I will explain why doing so is correct.



E is at the intersection of AB and CD
R is at the intersection of AD and BC
(ER is the focal plane in intersection with the plane of the quadrilateral)
BD in intersection with ER is G
Line a is perpendicular to ER in G
GN=GE; GM=GR; M,N are on line a
We build 2 circles: one through M,G,R, another one through N,E,G
The 2 circles have 2 common points: one is G, and the other one O is the center of the thin lens.
The thin lens is parallel to the focal plane (hence to ER)
OF=focal distance; OF is perpendicular to ER;
So we found everything about the thin lens.

Now I will explain why doing so leads us to the correct answer.



For this let's analyze how the image of an arbitary square ABCD forms using an arbitrary thin lens. (The thin lens is convergent because the image is real, and also because the image is real the square is farther than the focus from the lens).
I used the method of the auxiliary ray to build the image. (OG||AB; OG||CD; OE||AD; OE||BC; AC||ON)
As we can notice, for an arbitrary square G that is the intersection of A1B1 and C1D1
and E – the intersection of A1D1 and B1C1 lie in the focal plane. This is how I found the focal plane.
The angle GON and NOE have both the value of 45 degrees.
This part of the solution :
Line a is perpendicular to ER in G
GN=GE; GM=GR; M,N are on line a
We build 2 circles: one through M,G,R, another one through N,E,G
The 2 circles have 2 common points: one is G, and the other one O is the center of the thin lens.
Is just finding all the points Q that the angle NQE is 45 degrees and all the points W so that the angle NWG is also 45 grades. The intersection of the totality of points Q and W gives O – the center of the thin lens. (Q,W are of course, to the left of GE (the circles can be drawn also to the right of GE but O can't lie before the right focal plane)).
If we have the center of the lens and the focal plane, we just draw a line through O perpendicular to the focal plane and we have the orientation of lens, as well as the position.