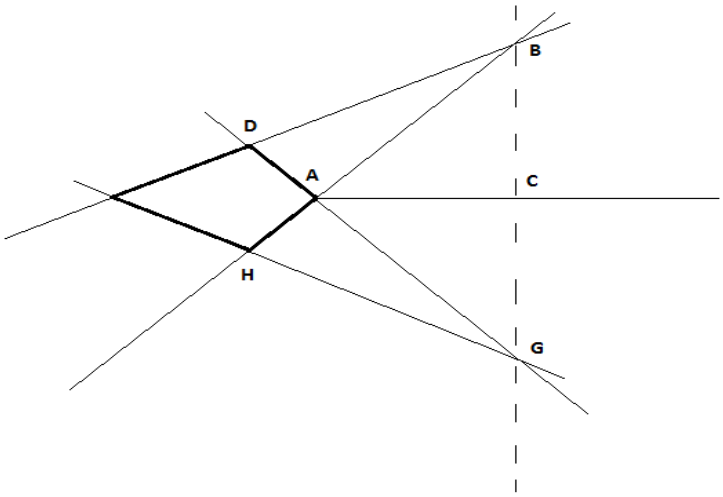
Problem no. 7(Solution)

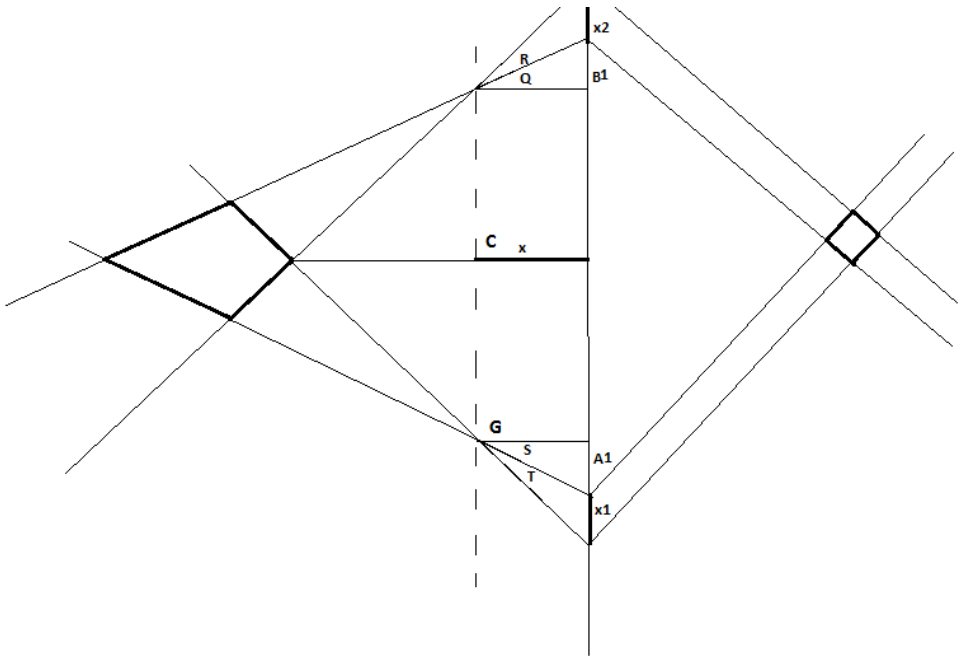
Petar Tadic, Gimnazija ,,Stojan Cerovic", Niksic
III grade

Let's start with following picture:



Here is how I have found the focal plane. Now I will suppose that numeric values of length of every side of image are known and also the lengths of the straight lines AC, AB, HG and DB.

Now I need to find out what angle has the beams after passing through the lens (angle with lens). Here is the second picture:



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From this picture I will write the conditions for having a square at right side:

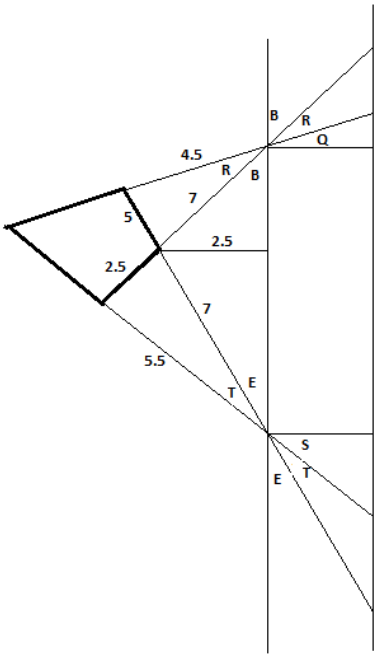
$$x_1 sinA_1 = x_2 sinB_1, \qquad tgA_1 = \frac{x_2}{x_1}. (A_1 + B_1 = 90)$$

The angles S, T, R and Q is also known. (They can be found using simple geometric laws and cosines theorem, from knowing the length of side of images and lines AC, AB, HG and DB on first picture.)

Now: $tg(R+Q) - tgQ = \frac{x_2}{x}, tg(S+T) - tgS = \frac{x_1}{x}$. From this: $\frac{x_2}{x_1} = \frac{tg(R+Q)-tgQ}{tg(S+T)-tgS}, tgA_1 = \frac{tg(R+Q)-tgQ}{tg(S+T)-tgS}$.

From these equations: $A_1 = arctg\left(\frac{tg(R+Q)-tgQ}{tg(S+T)-tgS}\right), B_1 = 90 - arctg(\frac{tg(R+Q)-tgQ}{tg(S+T)-tgS})$. Now it is just necessary to calculate the values of these two angles. After that we need to paint two hypothetical beams that pass through points B and G, on first picture, respectively, and have angles A and B with focal plane. (This one through point B will have angle B with focal plane, and this one through G will have angle A.) The point where they cut each other will be placed the center of lens and it must be parallel with focal plane.

Now I make a calculus:



This is the picture where I write the numerical values from the original image. The angles R and T can be found using the cosines law and they are: $R = 45°, T = 19°$, angles B and E are the same: $B = E = 21°$.

So $Q = 24°, S = 50°$. Now using the formula for angle of beams with lens, I get: $A_1 = 57°, B_1 = 33°$.

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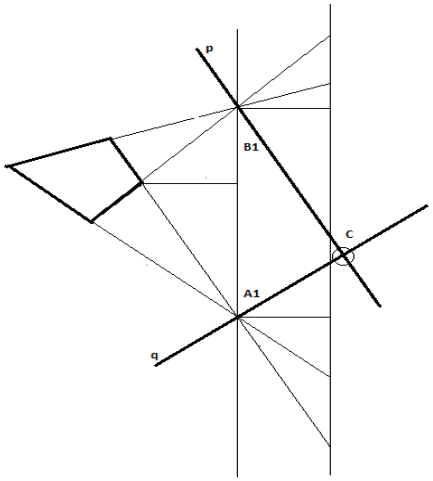

Here are two beams p and q with angles A1 and B1 with focal plane (and with lens, of course) The point C is location of center of lens.