Solution Problem No.4 (LONG VERSION)

$$\begin{cases} x_1 = x + Lsin(\theta_1) - Lsin(\theta_3) \\ x_2 = x + Lsin(\theta_2) - Lsin(\theta_3) \\ x_3 = x \end{cases} \qquad \begin{cases} y_1 = y + Lcos(\theta_1) - Lcos(\theta_3) \\ y_2 = y + Lcos(\theta_2) - Lcos(\theta_3) \\ y_3 = y \end{cases}$$

$$\begin{cases} \dot{x}_1 = \dot{x} + L\dot{\theta}_1 cos(\theta_1) - L\dot{\theta}_3 cos(\theta_3) \\ \dot{x}_2 = \dot{x} + L\dot{\theta}_2 cos(\theta_2) - L\dot{\theta}_3 cos(\theta_3) \\ \dot{x}_3 = \dot{x} \end{cases} \qquad \begin{cases} \dot{y}_1 = \dot{y} - L\dot{\theta}_1 sin(\theta_1) + L\dot{\theta}_3 sin(\theta_3) \\ \dot{y}_2 = \dot{y} - L\dot{\theta}_2 sin(\theta_2) + L\dot{\theta}_3 sin(\theta_3) \\ \dot{y}_3 = \dot{y} \end{cases}$$

$$\begin{cases} \dot{x}_1^2 = \dot{x}^2 + L^2\dot{\theta}_1^2 cos^2(\theta_1) + L^2\dot{\theta}_3^2 cos^2(\theta_3) + 2L\dot{x}\dot{\theta}_1 cos(\theta_1) - 2L\dot{x}\dot{\theta}_3 cos(\theta_3) - 2L^2\dot{\theta}_1\dot{\theta}_3 cos(\theta_1)cos(\theta_3) \\ \dot{x}_2^2 = \dot{x}^2 + L^2\dot{\theta}_2^2 cos^2(\theta_2) + L^2\dot{\theta}_3^2 cos^2(\theta_3) + 2L\dot{x}\dot{\theta}_2 cos(\theta_2) - 2L\dot{x}\dot{\theta}_3 cos(\theta_3) - 2L^2\dot{\theta}_2\dot{\theta}_3 cos(\theta_2)cos(\theta_3) \\ \dot{x}_3^2 = \dot{x}^2 \end{cases}$$

$$\begin{cases} \dot{y}_1^2 = \dot{y}^2 + L^2\dot{\theta}_1^2 sin^2(\theta_1) + L^2\dot{\theta}_3^2 sin^2(\theta_3) + 2L\dot{y}\dot{\theta}_3 sin(\theta_3) - 2L\dot{y}\dot{\theta}_1 sin(\theta_1) - 2L^2\dot{\theta}_1\dot{\theta}_3 sin(\theta_1)sin(\theta_3) \\ \dot{y}_2^2 = \dot{y}^2 + L^2\dot{\theta}_2^2 sin^2(\theta_2) + L^2\dot{\theta}_3^2 sin^2(\theta_3) + 2L\dot{y}\dot{\theta}_3 sin(\theta_3) - 2L\dot{y}\dot{\theta}_2 sin(\theta_2) - 2L^2\dot{\theta}_2\dot{\theta}_3 sin(\theta_2)sin(\theta_3) \\ \dot{y}_3^2 = \dot{y}^2 \end{cases}$$

$$\begin{cases} \ddot{x}_1 = \ddot{x} + L\ddot{\theta}_1 cos(\theta_1) - L\dot{\theta}_1^{\,2} sin(\theta_1) - L\ddot{\theta}_3 cos(\theta_3) + L\dot{\theta}_3^{\,2} sin(\theta_3) \\ \ddot{x}_2 = \ddot{x} + L\ddot{\theta}_2 cos(\theta_2) - L\dot{\theta}_2^{\,2} sin(\theta_2) - L\ddot{\theta}_3 cos(\theta_3) + L\dot{\theta}_3^{\,2} sin(\theta_3) \\ \ddot{x}_3 = \ddot{x} \end{cases}$$

$$\begin{cases} \ddot{y}_1 = \ddot{y} - L\ddot{\theta}_1 sin(\theta_1) - L\dot{\theta}_1^{\,2} cos(\theta_1) + L\ddot{\theta}_3 sin(\theta_3) + L\dot{\theta}_3^{\,2} cos(\theta_3) \\ \ddot{y}_2 = \ddot{y} - L\ddot{\theta}_2 sin(\theta_2) - L\dot{\theta}_2^{\,2} cos(\theta_2) + L\ddot{\theta}_3 sin(\theta_3) + L\dot{\theta}_3^{\,2} cos(\theta_3) \\ \ddot{y}_3 = \ddot{y} \end{cases}$$

$$\boldsymbol{T_i = \frac{1}{2} m_i(\dot{x}_i^2 + \dot{y}_i^2)}$$

$$\begin{cases} T_1 = \frac{1}{2}m[\dot{x}^2 + \dot{y}^2 + L^2\dot{\theta}_1^2 + L^2\dot{\theta}_3^2 + 2L\dot{x}\dot{\theta}_1 cos(\theta_1) - 2L\dot{x}\dot{\theta}_3 cos(\theta_3) + 2L\dot{y}\dot{\theta}_3 sin(\theta_3) - 2L\dot{y}\dot{\theta}_1 sin(\theta_1) - 2L^2\dot{\theta}_1\dot{\theta}_3 cos(\theta_3 - \theta_1)] \\ T_2 = \frac{1}{2}2m[\dot{x}^2 + \dot{y}^2 + L^2\dot{\theta}_2^2 + L^2\dot{\theta}_3^2 + 2L\dot{x}\dot{\theta}_2 cos(\theta_2) - 2L\dot{x}\dot{\theta}_3 cos(\theta_3) + 2L\dot{y}\dot{\theta}_3 sin(\theta_3) - 2L\dot{y}\dot{\theta}_2 sin(\theta_2) - 2L^2\dot{\theta}_2\dot{\theta}_3 cos(\theta_3 - \theta_2)] \\ T_3 = \frac{1}{2}3m[\dot{x}^2 + \dot{y}^2] \end{cases}$$

$$\boldsymbol{\sum_{i=1}^{i=3} T_i = \frac{1}{2} m\{6\dot{x}^2 + 6\dot{y}^2 + L^2(\dot{\theta}_1^2 + 2\dot{\theta}_2^2 + 3\dot{\theta}_3^2) + 2L\dot{x}[\dot{\theta}_1 cos(\theta_1) + 2\dot{\theta}_2 cos(\theta_2) - 3\dot{\theta}_3 cos(\theta_3)] + 2L\dot{y}[-\dot{\theta}_1 sin(\theta_1) - 2\dot{\theta}_2 sin(\theta_2) + 3\dot{\theta}_3 sin(\theta_3)] - 2L^2[\dot{\theta}_1\dot{\theta}_3 cos(\theta_3 - \theta_1) + 2\dot{\theta}_2\dot{\theta}_3 cos(\theta_3 - \theta_2)]\}}$$

$$\left(\frac{\partial \sum_{i=1}^{i=3} T_i}{\partial \dot{x}}\right) = \frac{1}{2} m\{12\dot{x} + 2L[\dot{\theta}_1 cos(\theta_1) + 2\dot{\theta}_2 cos(\theta_2) - 3\dot{\theta}_3 cos(\theta_3)]\}$$

$$\left(\frac{\partial \sum_{i=1}^{i=3} T_i}{\partial \dot{y}}\right) = \frac{1}{2} m\{12\dot{y} + 2L[-\dot{\theta}_1 sin(\theta_1) - 2\dot{\theta}_2 sin(\theta_2) + 3\dot{\theta}_3 sin(\theta_3)]\}$$

$$\left(\frac{\partial \sum_{i=1}^{i=3} T_i}{\partial \dot{\theta}_1}\right) = \frac{1}{2} m\{2L^2\dot{\theta}_1 + 2L\dot{x}cos(\theta_1) - 2L\dot{y}sin(\theta_1) - 2L^2\dot{\theta}_3 cos(\theta_3 - \theta_1)\}$$

$$\left(\frac{\partial \sum_{i=1}^{i=3} T_i}{\partial \dot{\theta}_2}\right) = \frac{1}{2} m\{4L^2\dot{\theta}_2 + 4L\dot{x}cos(\theta_2) - 4L\dot{y}sin(\theta_2) - 4L^2\dot{\theta}_3 cos(\theta_3 - \theta_2)\}$$

$$\left(\frac{\partial \sum_{i=1}^{i=3} T_i}{\partial \dot{\theta}_3}\right) = \frac{1}{2} m\{6L^2\dot{\theta}_3 - 6L\dot{x}cos(\theta_3) + 6L\dot{y}sin(\theta_3) - 2L^2[\dot{\theta}_1 cos(\theta_3 - \theta_1) + 2\dot{\theta}_2 cos(\theta_3 - \theta_2)]\}$$

$$\boldsymbol{\frac{d}{dt}\left(\frac{\partial \sum_{i=1}^{i=3} T_i}{\partial \dot{q}_i}\right) = 0} \qquad \dot{q}_i = \dot{q}_i(\dot{x}, \dot{y}, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3)$$

$$\frac{d}{dt}\left(\frac{\partial \sum_{i=1}^{i=3} T_i}{\partial \dot{x}}\right) = \frac{1}{2}m\left\{12\ddot{x} + 2L\left[\ddot{\theta}_1 cos(\theta_1) - \dot{\theta}_1^{\ 2} sin(\theta_1) + 2\ddot{\theta}_2 cos(\theta_2) - 2\dot{\theta}_2^{\ 2} sin(\theta_2) - 3\ddot{\theta}_3 cos(\theta_3) + 3\dot{\theta}_3^{\ 2} sin(\theta_3)\right]\right\} = 0$$

$$\frac{d}{dt}\left(\frac{\partial \sum_{i=1}^{i=3} T_i}{\partial \dot{y}}\right) = \frac{1}{2}m\left\{12\ddot{y} + 2L\left[-\ddot{\theta}_1 sin(\theta_1) - \dot{\theta}_1^{\ 2} cos(\theta_1) - 2\ddot{\theta}_2 sin(\theta_2) - 2\dot{\theta}_2^{\ 2} cos(\theta_2) + 3\ddot{\theta}_3 sin(\theta_3) + 3\dot{\theta}_3^{\ 2} cos(\theta_3)\right]\right\} = 0$$

$$\frac{d}{dt}\left(\frac{\partial \sum_{i=1}^{i=3} T_i}{\partial \dot{\theta}_1}\right) = \frac{1}{2}m\{2L^2\ddot{\theta}_1 - 2L^2\ddot{\theta}_3 cos(\theta_3 - \theta_1) + 2L^2\dot{\theta}_3(\dot{\theta}_3 - \dot{\theta}_1) sin(\theta_3 - \theta_1) + 2L\ddot{x} cos(\theta_1) - 2L\dot{x}\dot{\theta}_1 sin(\theta_1) - 2L\ddot{y} sin(\theta_1) - 2L\dot{y}\dot{\theta}_1 cos(\theta_1)\} = 0$$

$$\frac{d}{dt}\left(\frac{\partial \sum_{i=1}^{i=3} T_i}{\partial \dot{\theta}_2}\right) = \frac{1}{2}m\{4L^2\ddot{\theta}_2 + 4L\ddot{x} cos(\theta_2) - 4L\dot{x}\dot{\theta}_2 sin(\theta_2) - 4L\ddot{y} sin(\theta_2) - 4L\dot{y}\dot{\theta}_2 cos(\theta_2) - 4L^2\ddot{\theta}_3 cos(\theta_3 - \theta_2) + 4L^2\dot{\theta}_3(\dot{\theta}_3 - \dot{\theta}_2) sin(\theta_3 - \theta_2)\} = 0$$

$$\frac{d}{dt}\left(\frac{\partial \sum_{i=1}^{i=3} T_i}{\partial \dot{\theta}_3}\right) = \frac{1}{2}m\{6L^2\ddot{\theta}_3 - 6L\ddot{x} cos(\theta_3) + 6L\dot{x}\dot{\theta}_3 sin(\theta_3) + 6L\ddot{y} sin(\theta_3) + 6L\dot{y}\dot{\theta}_3 cos(\theta_3) - 4L^2\ddot{\theta}_2 cos(\theta_3 - \theta_2) + 4L^2\dot{\theta}_2(\dot{\theta}_3 - \dot{\theta}_2) sin(\theta_3 - \theta_2) - 2L^2\ddot{\theta}_1 cos(\theta_3 - \theta_1) + 2L^2\dot{\theta}_1(\dot{\theta}_3 - \dot{\theta}_1) sin(\theta_3 - \theta_1)\} = 0$$

Initial constraints: $\theta_1 \to \frac{\pi}{6};\qquad \theta_2 \to \frac{5\pi}{6};\qquad \theta_3 \to \frac{3\pi}{2};$

$$\begin{cases} \dot{x}_1 = \dot{x} + \frac{\sqrt{3}}{2}L\dot{\theta}_1 = 0 \\ \dot{x}_2 = \dot{x} - \frac{\sqrt{3}}{2}L\dot{\theta}_2 = 0 \\ \dot{x}_3 = \dot{x} = 0 \end{cases} \quad and \quad \begin{cases} \dot{y}_1 = \dot{y} - \frac{1}{2}L\dot{\theta}_1 - L\dot{\theta}_3 = 0 \\ \dot{y}_2 = \dot{y} + \frac{1}{2}L\dot{\theta}_2 - L\dot{\theta}_3 = 0 \\ \dot{y}_3 = \dot{y} = v_0 \end{cases} \qquad \ddot{y}_3 = \ddot{y} = 0$$

$$\begin{cases} 12\ddot{x} + \sqrt{3}L\ddot{\theta}_1 - 2\sqrt{3}L\ddot{\theta}_2 - 6L\dot{\theta}_3^{\ 2} = 0 \\ -L\ddot{\theta}_1 - 2L\ddot{\theta}_2 - 6L\ddot{\theta}_3 = 0 \\ \sqrt{3}L\ddot{x} + 2L^2\ddot{\theta}_1 + L^2\ddot{\theta}_3 - \sqrt{3}L^2\dot{\theta}_3^{\ 2} = 0 \\ -2\sqrt{3}L\ddot{x} + 4L^2\ddot{\theta}_2 + 2L^2\ddot{\theta}_3 + 2\sqrt{3}L^2\dot{\theta}_3^{\ 2} = 0 \end{cases} \Rightarrow \begin{cases} \ddot{x} = \frac{2}{11}\frac{v_0^2}{L} \\ \ddot{\theta}_1 = \frac{4\sqrt{3}}{11}\frac{v_0^2}{L^2} \\ \ddot{\theta}_2 = -\frac{5\sqrt{3}}{11}\frac{v_0^2}{L^2} \\ \ddot{\theta}_3 = \frac{\sqrt{3}}{11}\frac{v_0^2}{L^2} \end{cases}$$

$$\boldsymbol{a_i = \sqrt{\ddot{x}_i^2 + \ddot{y}_i^2}} \qquad \Rightarrow \begin{cases} a_1 = \frac{6}{11}\frac{v_0^2}{L} \\ a_2 = \frac{3}{11}\frac{v_0^2}{L} \\ a_3 = \frac{2}{11}\frac{v_0^2}{L} \end{cases}$$

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